

# Extended Spectral Proper Orthogonal Decomposition for Identifying the Correlated Structures in Building Surface Pressure and Surrounding Flow Fields

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## SUMMARY:

In this study, we propose a new analysis method named extended spectral proper orthogonal decomposition (ESPOD), which is a combination of the EPOD procedure and SPOD method, for identifying the correlated coherent structures in building surface pressure and surrounding flow fields. This method extracts velocity modes according to their phase synchronization with the SPOD pressure modes across the flow realizations. A case study of an isolated building reveals the main physical mechanisms of the wind forces, including the contributions of the approaching turbulence, wake vortices, and conical vortices. The results show that multiple physical mechanisms can occur at the same frequency and contribute differently to the wind forces. Therefore, neither frequency nor energy value can be used as a criterion for determining whether two structures in the velocity and surface pressure fields are caused by the same physical mechanism. However, their connection can be efficiently identified by the phase synchronization in ESPOD.

*Keywords: proper orthogonal decomposition, surface pressure, wind velocity*

## 1. INTRODUCTION

The proper orthogonal decomposition (POD) method has been used in numerous studies in wind engineering for extracting the key features from the surrounding flow and surface pressure of a building. Usually, POD is applied to the surface pressure field when the study aims to analyze building wind loads or responses. However, the surface pressure modes are less informative than the surrounding velocity field, resulting in difficulty in the interpretation of the modes. A question arises concerning whether we can find a connection between the pressure and velocity modes so that the corresponding velocity modes can help us better understand the surface pressure modes. To this end, this study combines the extended POD (EPOD) procedure (Borée, 2003) and the spectral POD (SPOD) method (Towne et al., 2018) to identify the correlated periodic structures, and the newly developed method is termed extended SPOD (ESPOD). The tasks of this study are to interpret the physical meanings of the surface pressure mode by the corresponding velocity mode and then quantify the contributions of the surrounding flow structures to the wind forces.

## 2. ESPOD AND WIND FORCE SPECTRA

The ESPOD method is based on the batch algorithm of SPOD (Towne et al., 2018), wherein the time history is divided into blocks, and a Fourier transform is conducted for each block. The resulting block data can be regarded as independent Fourier realizations for estimating a converged cross-spectral density matrix. Following this method, the  $l^{\text{th}}$  Fourier realization of the velocity  $\hat{\mathbf{u}}_f^{(l)}$  and surface pressure  $\hat{p}_f^{(l)}$  ( $l = 1, 2, \dots, N_b$ ) can be decomposed by the eigenmodes in the form of

$$\hat{\mathbf{u}}_f^{(l)}(\mathbf{x}_u) = \sum_{n=1}^{N_m} a_{f,n}(l) \boldsymbol{\varphi}_{f,n}(\mathbf{x}_u), \quad \text{and} \quad \hat{p}_f^{(l)}(\mathbf{x}_p) = \sum_{n=1}^{N_m} a_{f,n}(l) \psi_{f,n}(\mathbf{x}_p), \quad (1)$$

where the subscripts  $f$  and  $n$  denote the frequency and mode number.  $\mathbf{x}_u$  and  $\mathbf{x}_p$  are the spatial coordinates where the velocity and pressure are sampled.  $\boldsymbol{\varphi}$  and  $\psi$  are the modes of velocity and pressure, respectively. Ideally, the mode coefficients  $a$  are expected to be the same between velocity and pressure decomposition to ensure the phase synchronization between the corresponding modes. If this can be realized, the periodic flow motion indicated by a specified velocity mode will be the only part of the velocity field that is correlated to the periodic surface pressure variation indicated by the corresponding pressure mode (Borée, 2003), and vice versa. Unfortunately, the two decompositions cannot be both SPOD if the mode coefficients are required to be the same. Therefore, the strategy of EPOD provided by Borée (2003) is followed to extract the modes. Provided that the pressure mode is SPOD mode, the corresponding velocity mode can be computed as

$$\boldsymbol{\varphi}_{f,n}(\mathbf{x}_u) = \frac{1}{TN_b \lambda_{f,n}} \sum_{l=1}^{N_b} a_{f,n}^*(l) \hat{\mathbf{u}}_f^{(l)}(\mathbf{x}_u), \quad (2)$$

where  $T$  is the block length,  $N_b$  is the total number of the blocks (see Towne et al., 2018), and  $\lambda$  is the pressure mode energy. Notice that the velocity modes computed in this way are not orthogonal and thus do not optimize the energy, yet the velocity field can still be reconstructed by these modes.

In addition, the force spectra are defined to quantify the contributions of the modes on the three components of the base bending moment. The force spectrum vector is defined as

$$\boldsymbol{\Theta}_{f,n} = \lambda_{f,n} \left| \int_{\Omega_p} \psi_{f,n}(\mathbf{x}_p) (-\mathbf{x}_p \times \mathbf{n}) ds \right|^2, \quad (3)$$

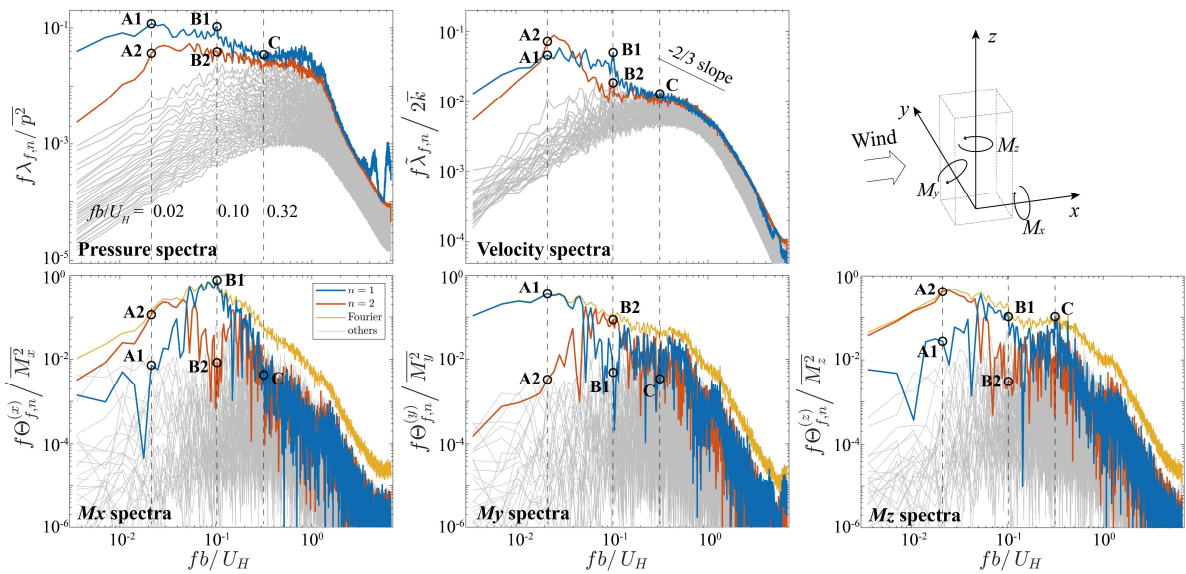
where  $\mathbf{n}$  is the normal vector on the building surface. It can be proved that if the component  $\boldsymbol{\Theta}_{f,n}^{(i)}$  is integrated over frequencies ( $f$ ) and summed over mode indices ( $n$ ), it becomes the variance of the bending moment component  $M_i$ .

### 3. CASE STUDY FOR AN ISOLATED BUILDING

The proposed analysis method was validated using a typical high-rise building case. The case was set according to Case A in the guidebook proposed by the Architectural Institute of Japan (AIJ) ([https://www.aij.or.jp/jpn/publish/cfdguide/index\\_e.htm](https://www.aij.or.jp/jpn/publish/cfdguide/index_e.htm)), wherein the building had a square cross-

section with the height being twice the width. A large-eddy simulation was conducted, and the time history data of the surrounding wind field and surface pressure were sampled. The Reynolds number, calculated using the building width and the mean wind speed at the building height at the inlet, was 31,000. The mean flow and second-order turbulent statistics were in good agreement with the experimental data provided by AIJ, and the surface pressure results were also validated against the experimental data in Tokyo Polytechnic University's database (<http://wind.arch.tokougei.ac.jp/system/eng/contents/code/tpu>). Please refer to Zhang et al. (2022) for the details of this simulation.

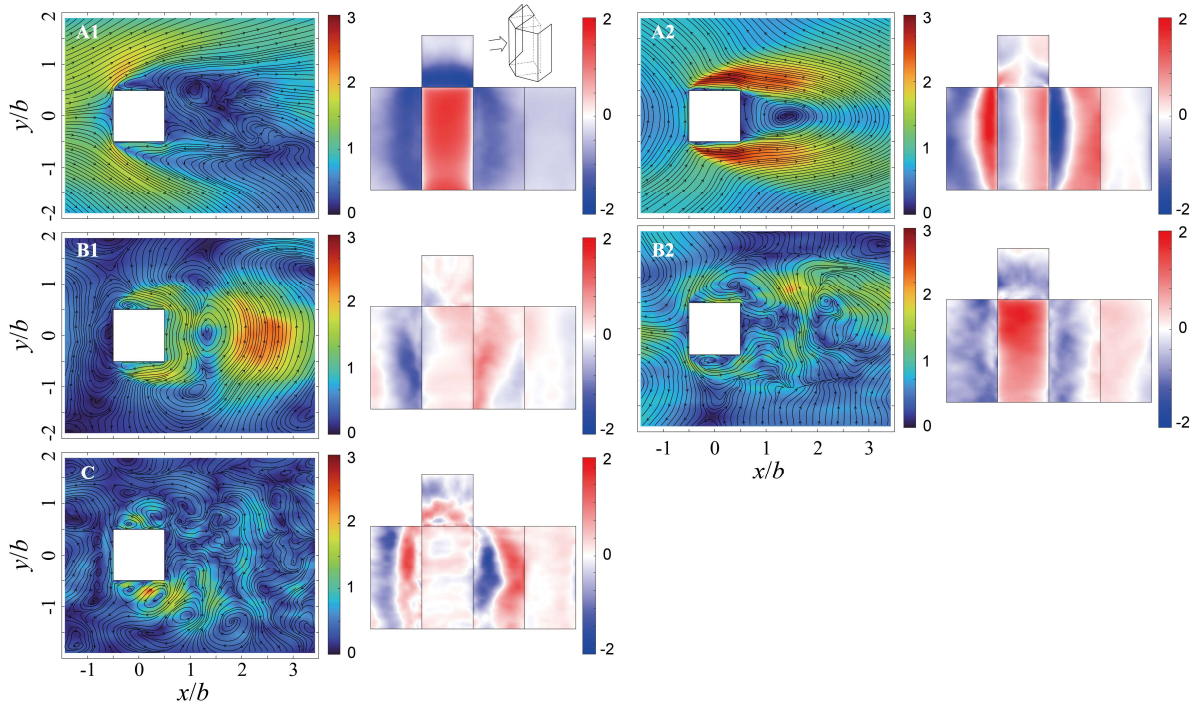
In the subsequent data analysis, the pressure modes were extracted by SPOD, and the velocity modes were extracted by ESPOD according to the phase synchronization with the pressure modes across the flow realizations. Figure 1 provides the energy spectra of the surface pressure field, velocity field, and three components of the base bending moment including cross-wind moment  $M_x$ , along-wind moment  $M_y$ , and torsional moment  $M_z$ . Each discrete point in the plots of spectra corresponds to a pair of periodic motions of the surface pressure and velocity fields. The periodic motions should be understood as the alternate appearance between the modes' real and imaginary parts. For example, the real parts of the modes at the circled points in Fig. 1 are provided in Fig. 2. For the wind force spectra, the Fourier spectra are also provided in Fig. 1, indicating the total energy at each discrete frequency.



**Figure 1.** Energy spectra of the surface pressure, velocity, and three components of the base bending moment.

At the low frequency of  $fb/U_H = 0.02$ , the periodic motions of surface pressure and surrounding flow are mainly caused by turbulence in the approaching flow. Both pressure and velocity modes at point A1 show patterns similar to the mean fields. This may be interpreted as the consequence of the low-frequency change in the intensity of the approaching flow. This symmetric mean-flow-like flow structure significantly affects the fluctuation of the along-wind moment. The pressure and velocity mode pair at point A2 exhibit the antisymmetric flow pattern caused mainly by the  $y$ -directional turbulence from the approaching wind. This flow pattern is the main contributor to the torsional moment fluctuation. The frequency of  $fb/U_H = 0.10$  is known as the Strouhal frequency

of the square cylinder. The most energetic mode (point B1) at this frequency in both velocity and pressure spectra illustrates the same vortex-shedding phenomenon, which is the major contributor to the cross-wind moment fluctuation. The second energetic mode at the same frequency (point B2) shows relatively small-scale symmetric components from the approaching flow, resulting in a minor along-wind moment fluctuation. Finally, the modes at the frequency of  $fb/U_H = 0.32$  (point C) show small-scale vortex shedding on the two sides of the building, which are also called conical vortices. These flow structures exhibit antisymmetric shapes, but it contributes mainly to the cross-wind moment fluctuation but little to the cross-wind one.



**Figure 2.** Selected SPOD pressure modes and ESPOD velocity modes (at  $z = b$ ). Only the real parts are shown.

#### 4. DISCUSSION AND CONCLUDING REMARKS

The case study of an isolated building shows that multiple physical mechanisms can occur at the same frequency and contribute differently to the kinetic energy and wind forces. Therefore, neither frequency nor energy value can be used as a criterion for determining whether two structures in the velocity and surface pressure fields are caused by the same physical mechanism. However, their connection can be efficiently identified by the phase synchronization in ESPOD.

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